

## **IMPROVING ANALYZING SKILLS OF PRIMARY STUDENTS USING A PROBLEM SOLVING STRATEGY**

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*This study makes use of an action research paradigm to improve primary students' analyzing skills. It was conducted at the San Esteban Elementary School, Region I, Philippines, during the 6-week off campus practice teaching of one of the researchers. Sources of data include a thinking skills checklist, a set of Curriculum Support Materials (CSM), field notes, and students' activity sheets. The study focused on using problem solving to improve and enhance the students' analyzing skills. Twenty-six (26) Grade Six students participated in this research. Findings reveal significant improvement in the students' analyzing skills after the use of problem solving in the classroom.*

### **INTRODUCTION**

One of the major objectives professed by most mathematics educators is to help students "learn to think." Thinking according to Marzano, Brandt, & Presseisen (1988) has various components such as focusing skills, information-gathering skills, remembering skills, organizing skills, analyzing skills, generating skills, integrating skills and evaluating skills. Bloom (1987) categorizes the first four as lower thinking skills and the rest as higher cognitive thinking skills. Of the higher cognitive thinking skills, analyzing

skill is directly associated to “doing mathematics” since this requires students to go beyond what they have done during previous instructions. Solving non-routine problems entails analysis, which on the other hand, requires students to discover and formulate relationships and to restructure the elements in a problem. Thorndike (in Sidhu, 1988) states that all the highest intellectual performance of the mind is analysis.

However, research shows that majority of students are poor in analysis. Ibe (cited in Martin 1999), noted that students could hardly solve problems that require the higher cognitive thinking skills. Likewise, the National Assessment of Educational Progress (as cited by French and Rhoder, 1992) reports that students failed to master higher, more abstract level of thinking such as analysis, synthesis and evaluation. Lumpas (1997) went further by saying the students can memorize, recall but not interpret, infer, judge or persuade.

All these make it imperative for the development of analyzing skills as one of the main goals of mathematics instruction. As a core thinking component, it should receive increased emphasis in the mathematics classroom. This, however, raises very important issues and questions like “How can one encourage students to perform tasks involving analysis?”, “How can students be involved in mathematics discourse that extends their understanding in analyzing?”

These two questions were primary considerations in the conduct of this study to improve students’ analyzing skills.

## **THE PROBLEM**

The Grade VI-B students of San Agustin elementary school can be considered as typical students in mathematics. They take the subject since it is required in the curriculum and they tackle activities given by the teacher for fear of a failing mark. The mathematics teacher reports that when they are given problems to solve, most start the solution but often stop midway and end up without an answer.

This happens especially when the problem requires more than just the application of a rule or algorithm. To understand the phenomenon better, classroom observations were conducted while the teacher implements the planned lesson for the day. Based on our observation notes, we inferred that poor analyzing skills of the students are due to the following:

- The traditional way of teaching—discuss, demonstrate, seatwork / boardwork, homework—provides minimal participation of students. This limits the development of their power of thinking.
- Few hands-on and minds-on activities that require students to explore, discover, make conjectures and reason logically are provided during instruction.
- Mathematical discourse is nearly absent in the classroom. Most often, the instruction is focused on finding the correct answer. Hence, they are not trained to think.
- The lack of teaching materials that allow students to pursue exploration and investigations.

The team discussed and deliberated on the findings and came up with a topic of interest: How can teaching practices be modified so that the students will develop and improve their analyzing skills?

The members conducted more observations and formulated questions related to the theme. After which, they decided to try out a strategy, which may not be new but has never been used extensively in the mathematics classroom, to enhance or improve the students analyzing skills.

## **SIGNIFICANCE OF THE STUDY**

In this era of information technology, our students are expected to exercise critical thinking and judgments, to analyze, evaluate and use information for effective decision making in their future jobs

and in their lives. Analyzing skills are used to clarify existing information by examining parts and relationships. Analysis helps us understand and think about the credibility of assumptions, observations, reasoning, and claims. These are all inherent in mathematics instruction. Thus, this study bears significance to both the teachers and the students. The teachers have the responsibility in establishing a learning environment that is conducive to the development and improvement of thinking. The experiences in the conduct of the study can help teachers understand better the importance of their actions in supporting or inhibiting students' thinking. Subsequently, this understanding can lead to provision of more relevant learning experiences for the students.

### **SCOPE AND LIMITATIONS OF THE STUDY**

The study focused on how to improve the analyzing skills of students in only one school in the Province of Ilocos Norte. Purposive and convenient sampling techniques were used in the choice of samples and locale of the study. The school was where one of the researchers was assigned for off-campus practicum. Problem solving is the only strategy employed to improve the students' analyzing skills. Other constraints, such as school activities, district meetings, have interrupted classes for long periods of time. This affected the students' interest and enthusiasm to the activities and lessons presented. The try-out teacher was also constrained to follow the content of the syllabus. Most of the tasks and activities involved measurement since this was the topic at the time this research was conducted.

## **REVIEW OF LITERATURE AND RELATED STUDIES**

### **Problem Solving in the Mathematics Classroom**

Charles and Lester (1982, p.5) state: "a problem is a task for which the person confronting it wants or needs to find a solution; the person has no readily available procedure for finding the solution; and the person must make an attempt to find a solution." Thus, in a problem solving activity there is an atmosphere of inquiry, investigation, and analysis of mathematical situations. It is an activity that provides a setting for the development of higher level of thinking such as justifying, generalizing, comparing, synthesizing, and analyzing (Hernandez, 1991). Krulick and Rudnick (1980) further described problem solving as a process by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of unfamiliar situations. In that, the individual must synthesize what he has learned and apply it to a new and different situation. To solve a problem is to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end that is not immediately attainable by appropriate means (Polya, 1949).

In 1989, the NCTM Curriculum and Evaluation Standards stated that problem solving should be an integral part of all mathematical activity (p. 23) and that students should use problem-solving approaches to investigate and understand mathematical concepts. According to Schroeder and Lester (1989) problem solving can be undertaken in the classroom using three approaches namely: teaching for problem solving, teaching about problem solving and teaching via problems solving. The goal for the first approach is for students to learn the concept and later apply this knowledge to problem solving situations. The second approach, teaching about problem solving, highlights Polya's model of problem solving so students become aware of their thinking processes while they are solving problems. The teacher who teaches about problem solving

also teaches the “heuristics” or “strategies” of problem solving. Meanwhile, in teaching via problem solving the teaching of concepts begins with a problem. The teacher emphasizes the value of problem solving as a means to learning mathematics. This complements the aspects of teaching for and about problems solving. These different types of problem solving are critical because of the potentially different impact these have on student achievement.

### **THINKING SKILLS AND PROCESSES**

Thinking is the active process by which we develop understandings of others, our world and ourselves. The process of thinking enables us to solve problems, interpret information, make sense of our feelings and attitudes, discuss important issues, establish beliefs, and work toward the completion of goals. There are a variety of theories and models reflecting different conceptions of thinking. For example, Marzano, Brandt and Presseisen (1988) set out that thinking can be considered an integration of metacognition, creative thinking, and critical thinking and that there are endless lists of thinking processes that draw on the core thinking skills. These processes include concept formation, principle formation, comprehending, problem solving, analyzing, decision-making, research, composing and oral disclosure. Concept formation, principle formation and comprehending are involved mainly in knowledge acquisition, while the problem solving, analyzing, decision-making, research and composing are involved in the production and application of knowledge. Oral discourse is a process for both producing and acquiring knowledge.

Thinking skills, on the other hand, are relatively discrete cognitive operations that can be considered the “building blocks” of thinking (Langrehr, 1990). Accordingly, the following core thinking skills are important for students to do, and can be taught and be reinforced in school: focusing skills, information-gathering skills, remembering

skills, organizing skills, analyzing skills, generating skills, integrating skills, and evaluating skills.

*Focusing skills* involve directing one's attention to selected information such as defining problems and setting goals. *Information-gathering skills* engage one to obtaining relevant data by observing and questioning. Storing and retrieving information are directly linked to *remembering skills* while *organizing skills* entail arranging information so it can be used more effectively. It also requires the use of skills in comparing, classifying and ordering or sequencing.

*Analyzing* clarifies existing information by identifying and distinguishing among components, attributes and other factors. Meanwhile, *generating skills* involve using prior knowledge to add new information beyond what is given. *Generating skills* lead to the construction of new ideas by using prior knowledge and linking it to existing structure to add meaning to new information. *Integrating* involves connecting and combining information to build new understandings. To be able to do this one should be capable in summarizing or abstracting information efficiently and restructuring existing knowledge to incorporate new information. *Evaluating* requires assessing the appropriateness and quality of ideas.

In Bloom's taxonomy, the thinking skills are classified as knowledge, comprehension, application, analysis, synthesis, and evaluation. The *knowledge* level involves information gathering in that it requires the skills of observation and recall of information, knowledge of dates, events, and places and of major ideas, and mastery of subject matter. *Comprehension* involves the demonstration of understanding and grasping the meaning of information, translating knowledge into new context; interpreting facts, comparing, contrasting; ordering, grouping, inferring causes; and predicting consequences. In *application*, one makes use of

knowledge, methods, concepts, and theories in new situations or solving problems using required skills.

*Analysis, synthesis* and *evaluation* are considered higher order thinking skills in Bloom's taxonomy. *Analysis* entails skills of taking apart components to see patterns, and recognize hidden meanings. In contrast, *synthesis* means putting together old ideas to create new ones. This may involve also relating knowledge from several areas, predicting, and drawing conclusions. *Evaluation* means judging the outcome. The skills demonstrated at this level are those of comparing and discriminating between ideas; assessing the value of theories, making choices based on reasoned arguments; and recognizing subjectivity.

## **IMPROVING THINKING**

The thinking skills renaissance in the 1980's has brought about debates and raised questions on how best to teach for developing students' intellectual prowess. Should thinking skills be taught directly or infused into the curriculum? How should it be assessed? How should effective thinking processes be defined, categorized and sequenced? How should effective thinking practices be manifested throughout a learning environment? What is the role of the teacher in shaping, reinforcing and improving students' thinking skills?

Beyer (1997) suggests that we can improve the quality of students' thinking by providing them with opportunities to engage in the kinds of thinking to be improved. These opportunities include *framing learning with thoughtful questions, provoking puzzlement or dissonance, engaging students in knowledge-producing activities, and structuring learning around knowledge-producing activities.*

A thoughtful question can stimulate or encourage students' thinking beyond the level of recall or translation. It requires students to use information they may not have encountered, or to restructure



information in order to produce something new. Perkins (1991) considers thoughtful questions as “thoughtfully demanding” because it requires the use of various kinds of complex thinking such as analysis, evaluation synthesis, and so on. We can consider these as effective questions that develop mathematical thinking.

The second strategy, provoking dissonance means putting students in situations in which they are confronted by something that bothers them. Wiggins (1987) considers these the “jolt” that can lead students to deeper understanding and which, may lead them to seek answers on their own. In problem solving, “dissonance” is akin to the “blockage” that a person must overcome to get to the solution of the problem. Engaging students in knowledge producing activities, on the other hand, consist of giving tasks that involve hypothesis making and testing or assessing the logic of an argument. This may also involve a more general, broader activity that incorporates a number of cognitive operations carried out over a longer period in the form of a project. These activities or tasks are expected to provide opportunities for students to engage in a variety of higher order thinking skills and opportunities for teachers to provide instruction for improving students’ execution of these skills.

## **RESEARCHES ON PROBLEM SOLVING**

There are researches that directly linked problem solving to students achievement in mathematics. Some of these studies looked into the impact of the type of problem solving used to learning. Erickson (1993) found that the students who were taught for problem solving had no gains in achievement in computation, application, or problem solving. On the other hand, achievement of students taught via problem solving significantly increased in each category. This indicates that the type of instructional approach may be a factor in raising student achievement. Likewise, Butkowski, Corrigan, Nemeth and Spencer (cited in Bay, 2000) found that third graders

who were explicitly taught problem solving strategies became better at using each strategy. Charles and Lester (1984), in their study of fifth and seventh graders found that students learning a process approach to problem solving had better achievement than their peers not exposed to the same approach. Hoffer and Gamoran (1993) in their study of the impact of various instructional approaches found that one of the three main determinants of student achievement was emphasis on problem solving, and this was particularly effective with low- and middle-ability groups. All these findings support the assumption that using problem solving in instruction does positively affect students' achievement in mathematics.

Much recent research in linking problem solving to achievement is related to problem-based curricula, that is, mathematics content presented in problem solving situations. Flowers and Kline (1998) found that fourth graders in a problem-based curriculum improved in skills, concepts, and problem solving. They attributed the gains in conceptual understanding to the fact that the curriculum materials used encourage invented strategies, which involves more reasoning and thinking on the part of the students. In seventh and eighth grades, students using the problem based curriculum and traditional textbooks were tested in several areas related to proportional reasoning (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1997). Students using the problem-based curriculum scored significantly higher on a proportional reasoning test in both their willingness to explain thinking and getting the answers right.

This sampling of studies provides evidence of the impact of problem solving not only to students' achievement in mathematics but also on their conceptual understanding and skills. This study attempts to find how problem solving can improve students' thinking skills especially on their ability to analyze.

## **METHODOLOGY**

The research made used of an action research paradigm in addressing the identified problem. Both quantitative and qualitative approaches were employed to gather data relevant to the problem.

The questions the researchers tried to answer were the following:

- What level of analyzing skills did the students exhibit before and after the use of problem solving?
- How did the strategy enhance the analyzing skills of the students specifically among the following dimensions: identifying attributes and components, identifying relationships and patterns, identifying main ideas and identifying errors?
- Is there a significant difference between students' analyzing skills before and after the intervention?

## **SAMPLES**

The research involved twenty-six (10 males, 16 females) Grade Six-B students of the San Agustin Elementary School, in the town of Bacarra province of Ilocos Norte, Philippines. These samples composed the class assigned to the student teacher when we conducted this study.

## **DATA GATHERING INSTRUMENTS**

The following instruments were main sources of data for this study:

*The Thinking Skills Checklist.* This contains indicators of analyzing skills that students exhibited while performing a mathematical task or problem. Twelve (12) skill indicators were included, following Marzano, Brandt, and Presseisen (1988) categorization of sub-skills for analysis. A Likert-scale ranging from 1 (poor) to 5 (outstanding) indicated the degree to which the pupils exhibited each of these

indicator. The checklist was used during observation to monitor and assess the students' level of analyzing skills.

*Curriculum Support Materials (CSM).* This package contains various lessons, tasks or activities that require students' to investigate, explore, and discover patterns and to solve problems of varying degrees of complexity. Materials in this package were designed specifically to enable students to demonstrate the different sub-skills under analysis. A set of questions that engages students' thinking accompanies each activity or task.

*Field notes.* These are observation notes written by the researchers while doing classroom observation.

*Activity Sheets.* include students' required output for the different activities and answers to the different problems. These provided t information on students' progress in their analyzing skills.

## **DATA GATHERING PROCEDURE**

Data gathering was done into two stages, the first of which was concentrated on determining students' strengths and weaknesses in doing analysis. For a week, students were observed while they performed the usual activities provided to them during instruction. The team separately prepared observation notes. Results of students' quizzes, seatwork/board work and answers to questions during recitation were all recorded and analyzed. These became the basis of their ratings in the thinking skills checklist. Their mean scores indicated the level at which each student exhibited the sub-skills included in the checklist.

The second stage of the study was conducted after the team did some consultations and deliberations on what changes should be implemented in the classroom. The CSM package was prepared and designed and was subjected to content and face validation. Since at this time the topic in the syllabus was on measurement, the

activities and problems prepared are related to areas and perimeter of plane figures. Each day a particular task or problem from the package was given to the students. These activity or task requires the students to engage in exploration, investigation, to reason mathematically, and to reflect. These were accompanied by a set of questions, which are meant to determine students' level of thinking. Students were observed while they are performing the required activities. The same checklist was used to rate their level of analyzing skills.

## FINDINGS AND DISCUSSION

Table 1 contains the mean ratings obtained by the samples for each indicator of analyzing skills before and after the use of problem solving. As aforementioned, we used the mean ratings to indicate the extent to which the students have exhibited the skill during the lesson and while performing a required activity.

As indicated in the table, the students' levels of analyzing skills before the use of problem solving are either *poor* or *fair*. In particular, the students have poorly exhibited the skills *correcting error in the presented solution* and *determining the main concept of the problem*. The lowest mean ratings were obtained in these two sub-skills. However, the students' levels of analyzing skills generally improved after the use of problem solving. In all indicators, a minimum mean increase of 0.4230 and a maximum mean increase of 1.4615 were noted after the intervention. The sub-skills *correcting errors in the presented solution* and *determining the main concept in a problem* remain to be the lowest rated skill in the checklist. This indicates that among the sub-skills under analyzing skills these two seem to pose the greatest problem among the students.

Table 1  
*Students' obtained mean ratings before and after the use of problem solving*

Indicators of Analyzing Skills	Mean (Before)	Mean (After)
<b>A. Identifying Attributes</b>		
• Identifying what are given and what are asked for	2.4616	3.5384
• Systematically arranging data through tables, diagrams, graphs, or charts, & others	2.1153	2.8076
• Recognizing relevant and irrelevant data	1.8846	3.000
• Labeling correctly the parts of a figure	2.0384	3.0769
<b>B. Identifying Relationships and Patterns</b>		
• Finding patterns	1.7692	2.6538
• Recognizing relationships	1.6923	3.1538
• Making equations for a general case	1.8461	3.1153
• Relating obtained results to the original problem	1.8076	3.0769
<b>C. Identifying Errors</b>		
• Recognizing mistakes in calculation and procedures	1.6538	2.7307
• Correcting errors in the presented solution	1.5384	2.5384
<b>D. Identifying the Main Ideas</b>		
• Stating the rule or equation to represent the given conditions in the problem	1.6538	2.7307
• Determining the main concepts of the problem	1.5000	1.9230

*Legend:*

4.55 – 5.00(Outstanding) 3.55 – 4.54 (Very Good) 2.55 – 2.54 (Good) 1.55 – 2.54 (Fair) 1.00 – 1.54(Poor)

The t-test was used to test the assumption that a significant difference exist between the students’ level of analyzing skills before and after the intervention. For this purpose, the cluster mean was obtained and the level of significance is set at 0.05. Table 2 shows the result.

Table 2  
*t-test for mean difference*

No. of Samples	Cluster Mean	D	t – value
26	Before	1.7691	0.9973
	After	2.7667	

Critical t – value: 3.182                       $p < 0.05$

Based on the analysis, it can be said that there is a significant difference between the students’ level of analyzing skills before and after the intervention. The obtained t-value is significant at the 0.01 level. This means, that the increase in the ratings maybe due to problem solving 99% of the time.

## RESULTS FROM STUDENTS’ WORKSHEETS

Students’ output and responses give qualitative evidences that support the quantitative results obtained. Moreover, these reveal the remarkable impact of problem solving to their analyzing skills. The following improvements in students’ skills were gathered from extensive analyses of students’ output as they were exposed to the intervention:

- Students have become more organized and logical in setting-up a problem. They use more often the problem solving heuristics and rewrite problems in their own words to facilitate understanding and gain insights about the given task.

- They became skilled at organizing data using different tools such as diagrams, sketches, tables, and charts. Whenever they are confronted with a new task, the students are able to decide on how best they could organize the data on hand when *listing the given data* can not help them understand the problem.
- They show ability to break problems into simpler parts.
- Students show understanding of how parts of something fit together, recognize ways how things are related and use these in finding what is required in the problem.
- They are able to make a model or representation of a situation or relationships.

As observed, students become more analytical and logical in their thinking as they were exposed constantly and continuously to problem solving. Students not only gain skills in identifying given conditions but they became adept in formulating models, representations, and equations to describe a mathematical situation.

## **CONCLUSIONS AND RECOMMENDATIONS**

The findings from this research show that:

- The students have marginally poor analyzing skills. However, an improvement in their level of analyzing skills was noted after the use of problem solving;
- Problem solving can effectively improve students' skills in identifying attributes & components, relationships, patterns, errors and main ideas of the problem. The strategy encourages students to make conjectures, build connections among ideas, and conjecturing. This is shown from the checklist and students' worksheets.
- A significant difference exists between the students' level of analyzing skills before and after the use of problem solving skills.



This study lends support to previous results linking problem solving to favorable instructional outcomes. The results confirm earlier findings that an innovative strategy can improve student performance in mathematics. The following recommendations merit attention:

- Organize instruction around problem solving focusing on developing and strengthening students' skills in reasoning, representing ideas and data using different formats, making connections to other mathematical ideas and communicating their ideas to others.
- Place considerable importance on exploratory activities, observation and discovery since these build their confidence and a sense of responsibility for their own learning. This also facilitates their understanding of mathematical concepts.
- There is a need for teachers to use more non-traditional approaches in teaching mathematics for these make learning more relevant and enriching on the part of the students. Traditional approaches limit the development of students' mathematical thinking since they are simply receivers of knowledge.

## **IMPLICATIONS**

This study has shown that non-traditional teaching strategies such as problem solving can contribute significantly to developing and improving thinking skills. These can provide students a context for learning mathematical knowledge and enhance transfer of skills to new and unfamiliar situations. This can help teachers view many aspects of their teaching in a new perspective. The research can encourage teachers to find and try other teaching strategies to help their students become critical thinkers.

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## APPENDICES

### Sample Tasks and Sample Solutions

**Problem:** How many different rectangles with an area of 24 sq. cm. can be formed from a ribbon 120 cm. long?

#### Sample Solution

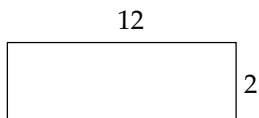
Asked: The number of different rectangles of area 24 sq. cm. from a ribbon 120 cm long

Given: length of ribbon = 120 cm

A of rectangle = 24 sq. cm.

$24 = 12 \times 2 = 6 \times 4 = 8 \times 3 \Rightarrow$  length and width of the rectangle

Length	Width	Total
12	2	$12 + 12 + 2 + 2 = 28$
6	4	$10 \times 2 = 20$
8	3	$11 \times 2 = 22$
Total		70

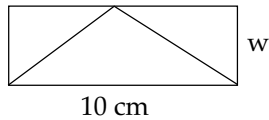


*There are 70 cm used. There are 50 cm more. 28 & 22 make 50*

*Answer: two 12 x 2 rectangle  
Two 8 x 3 rectangle  
One 6 x 4 rectangle*

Another group of students answered there are only 3 different rectangles since the others are the same (referring to two rectangles with the same dimensions such as  $12 \times 2$  and  $2 \times 12$ ).

**Problem:** The area of the triangle below is 25 sq. cm. Find the area of the rectangle.



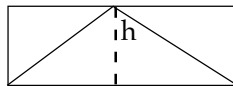
**SOLUTION**

The base of the triangle is 10 cm. it is also the length of the rectangle.

The area of the rectangle is :  $A = l \times w$

The area of the triangle is:  $A = 1/2 bh = 25$  sq. cm.

Diagram:



$h$  = height of the triangle

$h = w$ , it is not given

Since  $A = 25 = 1/2 bh$  and  $b = 10$ , I get  $10h/2 = 25$  or  $5h = 25$ .

$$5 \times 5 = 25, \text{ so } h = 5.$$

So  $w = 5$  because they are equal.

$$A = l \times w = 10 \times 5 = 50.$$

Answer: The area of the rectangle is 50 sq. cm.